LETTER TO THE EDITOR

Comment on "Cross-Sectional Shape of Collapsible Tubes"

Dear Sir:

The object of this note is to point out that some of the theoretical results of Kresch and Noordergraaf (1972) can be calculated by the theory of von Mises (1914). Kresch and Noordergraaf have computed the cross-sectional area and compliance of a long, thin-walled tube as a function of its unstressed elliptical cross-section and of its transmural pressure. Their Figs. 10 and 13 *b* show that compliance is maximum when the tube collapses. At the maximum of compliance, the normalized transmural pressure $P_{\max}$ has a value of approximately -2.

According to the theory of von Mises, a cylindrical tube will collapse when the transmural pressure is less than a critical value p_{crit}. We will take p_{crit} as a measure of Kresch and Noordergraaf's $p_{\max}$. For a tube that is long compared with its radius von Mises's p_{crit} reduces to (Timoshenko and Gere, 1961)

$$p_{\text{crit}} = -\frac{Eh^3(n^2-1)}{12a^3(1-\nu^2)},$$

where is E is Young's modulus, h the wall thickness, a the radius of the cylinder, ν Poisson's ratio, and n the number of circumferential lobes in the collapsed tube. In terms of Kresch and Noordergraaf's normalization, the normalized critical pressure P_{crit} is

$$P_{\text{crit}} = \frac{6p_{\text{crit}}}{Eh^3} = -\frac{(n^2-1)}{2a^3(1-\nu^2)}.$$

To calculate P_{crit} we note that there are two lobes in Kresch and Noordergraaf's theory at collapse or $n = 2$ and that the cross-section is circular with $a = 1$ when their elliptical eccentricity parameter $k = 1$. If we assume that the material of the wall is incompressible, $\nu = 1/2$. For these values of the parameters $P_{\text{crit}} = -2$. If $\nu = 0.43$, then $P_{\text{crit}} = -1.85$, which is equal to the value of $P_{\max}$ found by Kresch and Noordergraaf for a cylinder. For larger values of k the unstressed cross-section is no longer circular, but the value of $P_{\max}$ does not vary more than $\pm 10\%$ from -2. The fact that Kresch and Noordergraaf's calculation of $P_{\max}$ agrees with the calculation of P_{crit} according to the classical theory of von Mises lends weight to their other calculations which cannot be checked so readily.

Experimental evidence for identifying p_{crit} with $p_{\max}$ can be obtained from the work of Conrad (1969) and Moreno et al. (1970) on horizontal, thin-walled, rubber tubes. Conrad measured the pressure-volume characteristic and compliance of a tube with the following properties: $a = 0.63$ cm, $h = 0.093$ cm, $E = 1.6 \times 10^6$ dynes/cm^2, length $= 8.9$ cm. Moreno et al. measured the pressure-volume characteristics of a tube with $a = 0.62$ cm, $h = 0.053$ cm, $E = 21 \times 10^6$ dynes/cm^2, length $= 29$ cm. Assuming $\nu = ½$, for Conrad's tube $p_{\text{crit}} = -1.3$ mm Hg, while the measured value of $p_{\max}$ was -0.5 mm Hg. For the tube of Moreno et al. $p_{crit} = -3.2$ mm Hg and $p_{max} = -2.1$ mm Hg. The difference between the theoretical p_{crit} and the experimental p_{max} is not great and can be attributed in part to the effect of gravity on the wall and fluid, to longitudinal tension or to flexing[1] of the unstressed tube. All of these factors cause transverse forces which add to that due to transmural pressure and thus reduce the threshold for collapse. Since the experiments of Moreno et al. show that p_{max} for excised veins is positive, gravity and longitudinal tension cannot be ignored when calculating the conditions for collapse of veins. Flexing may also play a role.

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[1] Pressurized straight or curved tubes acted upon by end bending moments will collapse from this cause alone. For a comprehensive theory of collapse of tubes due to flexing and for references to earlier work, see Reissner (1959). In the body flexing of blood vessels takes place at joints and between points of unequal support.